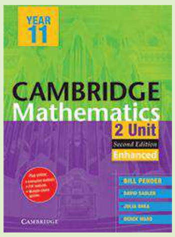
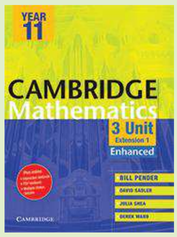
NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS ADVANCED (INCORPORATING EXTENSION 1) YEAR 11 COURSE

Topic summary and exercises:

With references to





A X1 Coordinate Geometry

Name: ....

Initial version by H. Lam, April 2016. Last updated March 27, 2021.
Various corrections by students & members of the Mathematics Department at Normanhurst Boys High School.

Symbols used

- Beware! Heed warning.
- Review
- Provided on NESA Reference Sheet.
- Mathematics Advanced content.
- Mathematics Extension 1 content.
- Literacy: note new word/phrase.
- Understanding (as opposed to memorisation) required!

$\mathbb{N}$ the set of natural numbers

$\mathbb{Z}$ the set of integers

$\mathbb{Q}$ the set of rational numbers

$\mathbb{R}$ the set of real numbers

$\forall$ for all

Syllabus outcomes addressed

MA11-1 uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems

MA11-2 uses the concepts of functions and relations to model, analyse and solve practical problems

ME11-1 uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses

ME11-2 manipulates algebraic expressions and graphical functions to solve problems

Syllabus subtopics

MA-F1 Working with Functions

MA-F2 Graphing Techniques

ME-F1 Further Work with Functions

Gentle reminder

- For a thorough understanding of the topic, *every* question in this handout is to be completed!
- Additional questions from *Cambridge Year 11 3 Unit* will be completed at the discretion of your teacher.
- Remember to copy the question into your exercise book!

Contents

Section 1

Points and intervals

Learning Goal(s)

Knowledge
The distance and midpoint formula in the Cartesian plane

Skills
Finding the distance and midpoint of an interval

Understanding
The derivation of the distance and midpoint formulas

By the end of this section am I able to:

5.1 Distance formula between two points in the Cartesian Plane

5.2 The midpoint formula

1.1 Review of formulae

Laws/Results

The distance formula to calculate the distance between two points (x_1, y_1) and (x_2, y_2):

$$d = \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

Derivation of formula:

Laws/Results

The midpoint formula to calculate the midpoint between two points (x_1, y_1) and (x_2, y_2):

$$M(x, y) = \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

Derivation of formula:

Example 1

The interval joining $A(3, -7)$ and $B(-6, 2)$ is a diameter of a circle. Find the centre and radius of the circle.

Answer: $C\left(-\frac{3}{2}, -\frac{5}{2}\right)$, $r = \frac{9}{2}\sqrt{2}$

Further exercises

2 **Ex 6A**

- Q1-2 last column
- Q3-10
- Q16-17

Section 2

Gradient

Learning Goal(s)

Knowledge
The relation between parallel and perpendicular lines

Skills
Determine the type of quadrilateral by examining the gradient of its sides and diagonals

Understanding
The relation between the gradient and the angle of inclination

By the end of this section am I able to:

5.3 Calculate the gradient of an interval

5.4 Examine and use the relationship between the angle of inclination of a line or tangent with the positive x-axis, and the gradient m of that line or tangent, and establish that tan $= m$

5.5 Understand and use the fact that parallel lines have the same gradient and that two lines with gradient m_1 and m_2 respectively are perpendicular if and only if $m_1 m_2 = -1$

5.6 Test for the special quadrilaterals in the Cartesian plane using distance, midpoint and gradient calculations

2.1 Review of formulae

Laws/Results

The gradient formula to calculate the gradient between two points (x_1, y_1) and (x_2, y_2):

$$m = \dots\dots\dots\dots\dots\dots$$

Derivation of formula:

Laws/Results

The angle of inclination θ of the line and the positive direction of the x axis:

$$m = \dots\dots\dots\dots$$

Derivation of formula:

Laws/Results

Parallel lines have the .... gradient.

Laws/Results

The gradient of two perpendicular lines are .... :

.... = ... or ....

Example 2

Show that the points $A(-2,5)$, $B(1,3)$ and $C(7,-1)$ are collinear

Example 3

[Ex 5B Q16] Answer the following for the points $W(2,3)$, $X(-7,5)$, $Y(-1,-3)$ and $Z(5,-1)$:

(a) Show that WZ is parallel to XY.

(b) Find the lengths WZ and XY. Hence deduce the type of quadrilateral $WXYZ$.

(c) Show that that the diagonals WY and XZ are perpendicular.

Example 4

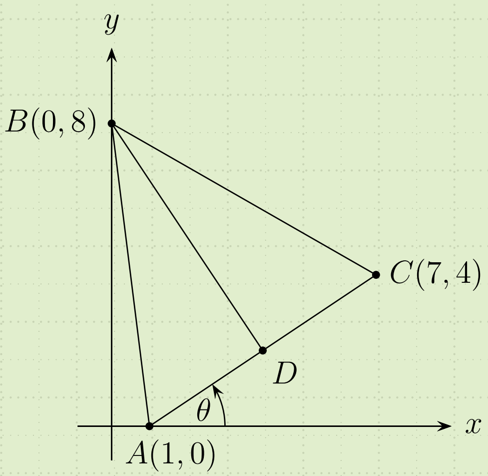
[Ex 5B Q18]

(a) $A(1,4)$, $B(5,0)$ and $C(9,8)$ form the vertices of a triangle. Find the coordinates of P and Q if they divide the sides AB and AC respectively in the ratio $1:3$.

(b) Show that PQ is parallel to BC and is one quarter of its length.

Example 5

The points A, B and C have coordinates $(1,0)$, $(0,8)$ and $(7,4)$, and the angle between AC and the x axis is θ.



(a) Find the gradient of the line AC and hence determine θ to the nearest degree. **2**

(b) Find the equation of AC. **2**

(c) Find the coordinates of D, the midpoint of AC. **2**

(d) Show that $AC \perp BD$. **1**

(e) What type of triangle is ABC? Show full reasoning. **2**

(f) Find the area of this triangle. **2**

(g) Write down the coordinates of the point E such that $ABCE$ is a rhombus. **2**

Further exercises

(2) **Ex 6B**

- Q7-21 odd #

(x1) **Ex 5B**

- Q7-21 odd #

Section 3

Equation of straight line

Learning Goal(s)

Knowledge
The different forms of straight lines

Skills
Finding the equation of straight lines

Understanding
Applying the appropriate form of a straight line to find its equation

By the end of this section am I able to:

5.7 Know and use the following forms of a straight line: gradient-intercept form $y = mx + c$, and general form $ax + by + c = 0$

5.8 Derive the equation of a straight line passing through two points (x_1, y_1) and (x_2, y_2) by first calculating its gradient m using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

5.9 Derive the equation of a straight line passing through two points (x_1, y_1) and (x_2, y_2) by first calculating its gradient m using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

5.10 Determine if three distinct lines are concurrent

5.11 Find the equations of straight lines, including parallel and perpendicular lines, given sufficient information

3.1 Review of formulae

Laws/Results

The **gradient-intercept form** of the equation of a line:

....

where

- ...... – ....
- ... – ....

Laws/Results

The **general form** of the equation of a line:

....

Laws/Results

The **point-gradient formula** to find the equation of a straight line through (x_1, y_1) with a given gradient m:

$$\dots\dots\dots\dots\dots\dots\dots\dots$$

Derivation of formula:

Important note

No new theory is in this section. However, expect slightly more difficult problems within textbook exercises.

Example 6

[**Ex 5D Q6**] Given the points $A(1, -2)$ and $B(-3, 4)$, find in general form the equation of:

(a) the line AB,

(b) the line through A perpendicular to AB.

Example 7

[Ex 5D Q12]

(a) On a number plane, plot the points $A(4,3)$, $B(0,-3)$ and $C(4,0)$.

(b) Find the equation of BC.

(c) Explain why $OABC$ is a parallelogram.

(d) Find the area of $OABC$ and the length of the diagonal AB.

Further exercises

2 **Ex 6C**

- Q5, 10-16

2 **Ex 6D**

- Q5
- Q7 last 3 columns
- Q14, 18-20

x1 **Ex 5C**

- Q6, 10-14

x1 **Ex 5D**

- Q4, 7-10, 13-18

Section 4

General coordinate geometry

Learning Goal(s)

Knowledge
The applications of the distance, midpoint and gradient formulas in geometry

Skills
Solve problems in the Cartesian plane

Understanding
The appropriate formulas and techniques to apply in solving geometric problems in the Cartesian plane

By the end of this section am I able to:

5.12 Solve a range of geometric problems using coordinate geometry concepts, including problems using pronumerals (rather than specific numeric values)

Further exercises

2 **Ex 6G**

- Q1-9

X1 **Ex 5G**

- Q1-14

Legacy 2 Unit course content - obsolete from Year 11 2019

Section A

② Perpendicular distance from a point to a line

Important note

From the legacy Mathematics 2 Unit course. Provided for interest, and also so that some of the legacy HSC examination questions can be attempted.

A.1 Results and formulae

- The shortest distance from a point to a line (or any object) is the .... distance.

Laws/Results

The **perpendicular distance formula** to calculate the distance from a point (x_1, y_1) to the line $ax + by + c = 0$:

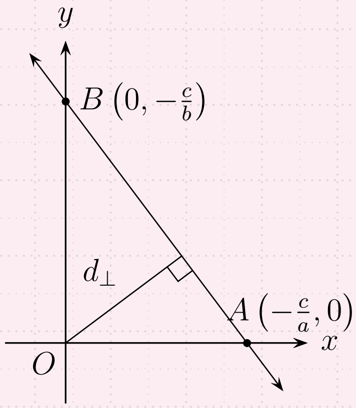
$$d_\perp = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Derivation of formula – Calculate the perpendicular distance from the origin to a line $ax + by + c = 0$, then shift origin to another point (x_1, y_1) along with the line.

Steps

1. Find the lengths of OA and OB:

- $OA =$ ....
- $OB =$ ....



2. Calculate the area of $\triangle OAB$ from OA and OB:

3. Calculate the area of $\triangle OAB$ from AB and length $d_\perp$:

4. Equate the two area expressions:

5. Shift the origin to another location $P(x_1, y_1)$ along with the line $ax+by+c = 0$:

Example 8

Find the perpendicular distance of $P(-2,5)$ from $y = 2x - 1$. **Answer:** $2\sqrt{5}$

A.2 Circles, chords, tangents and perpendicular distance

Theorem 1

The **tangent** to a circle is **perpendicular to** the **radius** drawn to the point of contact.

Laws/Results

A line will

- .... the circle .... if it is a .... or ....
- .... the circle .... at the point of contact if it is a ....
- .... the circle entirely if $d_\perp$ to the .... of the circle is greater than the ....

Draw situations presented by the three cases.

Example 9

For the circle $(x-7)^2+(y-6)^2=25$:

(a) Show that $3x+4y-20=0$ is a tangent to the circle.

(b) Find the length of the chord of this circle cut off by the line $3x+4y-60=0$.

Answer: 8 units

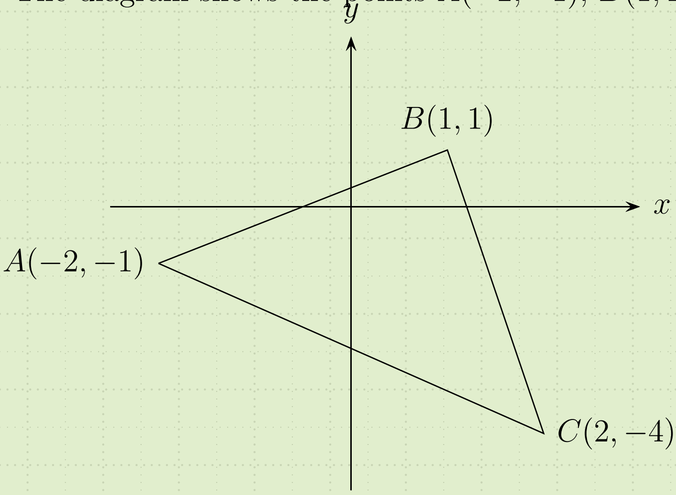
Example 10

For what value(s) of k will the line $5x-12y+k=0$ never intersect the circle with centre $P(-3,1)$ and radius 6?

Answer: $k<-51$ or $k>105$

Example 11

[2011 CSSA] The diagram shows the points $A(-2,-1)$, $B(1,1)$ and $C(2,-4)$.



(i) Calculate the length of the interval AB. **1**

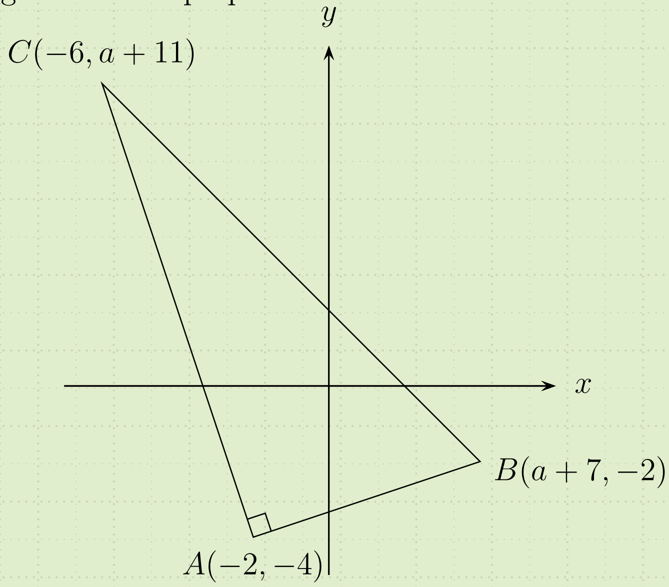
(ii) Find the equation of the line AB. **2**

(iii) Show that the perpendicular distance from C to the line AB is $\dfrac{17}{\sqrt{13}}$. **1**

(iv) Hence, calculate the area of $\triangle ABC$. **1**

Example 12

The points A, B and C have coordinates $(-2,-4)$, $(a+7,-2)$ and $(-6,a+11)$ as shown in the diagram. AB is perpendicular to AC.



NOT TO SCALE

(a) Find the gradient of AB and AC in terms of a. **2**

(b) Show that the coordinates of the points B and C are $(4,-2)$ and $(-6,8)$ respectively. **2**

(c) Find the equation of the line ℓ passing through A and perpendicular to BC. **2**

(d) The two lines ℓ and BC intersect at point D. Find the coordinates of D. **2**

(e) X1 Show that the equation of the circle passing through the points A, D and C is $x^2+y^2+8x-4y-20=0$. **2**

(f) Draw a neat sketch on a number plane showing A, B, C, the line AD and the circle. **1**

(g) A point E lies on the circle such that $EADC$ is a rectangle. Find the coordinates of E. **1**

Further exercises

② **Ex 6E**
- All questions

X1 **Ex 5E**
- Q2 onwards

Section B

② Line through the intersection of other lines

Important note

From the legacy Mathematics 2 Unit course.

It is provided here for interest, and also so that some of the legacy HSC examination questions can be attempted. Most of the past HSC questions will no longer be relevant in the new calculus course HSC (first HSC examination in 2020).

Laws/Results

'k' method If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ intersect at $M(x_0, y_0)$, then every other line that passes through $M(x_0, y_0)$ will have the form

....

for various values of k.

GeoGebra

Geogebra construction/demonstration:

1. Plot the following lines:

$$\begin{cases} 2x + 3y + 4 = 0 \\ 3x - 2y + 1 = 0 \end{cases}$$

2. Create a new slider 'k' with values ranging from -10 to 10.

3. Create the line through the intersection of the two previous lines:

....

4. What does the line trend towards as $k \to \pm\infty$?

5. Swap where the k value appears. What does the line trend towards as $k \to \pm\infty$?

Example 13

(a) Write down the equation of a line through the intersection M of the lines

$$\begin{cases} \ell_1 : & x + 2y - 6 = 0 \\ \ell_2 : & 3x - 2y - 6 = 0 \end{cases}$$

(b) Hence without finding the point of intersection, find the line through M that

i. passes through $P(2, -1)$.

ii. has a gradient of 5

iii. is horizontal

iv. is vertical

Answer: i. $5x - 2y - 12 = 0$ ii. $10x - 2y - 27 = 0$ iii. $y = \frac{3}{2}$ iv. $x = 3$

Example 14

[Ex 5F Q7]

(a) Write down the general form of a line through the intersection M of $x - 2y + 5 = 0$ and $x + y + 2 = 0$. Show that the gradient of this line is $\dfrac{1+k}{2-k}$.

(b) Hence find the equation of the lines through M:

i. parallel to $3x + 4y = 5$

ii. perpendicular to $2x - 3y = 6$

iii. perpendicular to $5y - 2x = 4$

iv. parallel to $x - y - 7 = 0$

Further exercises

(2) **Ex 6F**

- Q2-9

(x1) **Ex 5F**

- Q1-13

Section C

② Regions

C.1 Simple regions

Laws/Results

A region in the number plane is formed whenever a graph is drawn.

Example 15

(a) Sketch the line $y = 3x + 4$.

(b) Shade in the region where $y > 3x + 4$.

Important note

- Check boundary!
- This is *not* a number line!

Example 16

Sketch the region $x^2 + y^2 \leq 4$.

Example 17

Sketch the region $y > x^2$.

Example 18

Sketch $y \leq \dfrac{1}{x}$.

Example 19

Sketch $y < \sqrt{25 - x^2}$.

C.2 Compound regions

Definition 1

The *union* of two regions A and B is the region covered by either A *or* B.

Definition 2

The *intersection* of two regions A and B is the common region covered by A *and* B.

Example 20

Graph the region represented by

1. $\begin{cases} y > x^2 \\ x + y \leq 2 \end{cases}$ and

2. $\begin{cases} y > x^2 \\ x + y \leq 2 \end{cases}$ or

Example 21

Graph the region bounded by $x^2 + y^2 \leq 25$, $x \leq 3$ and $y > -4$.

Further exercises

2 Ex 4G

- Q1-15

X1 Ex 3F

- Q2-3 last 2 columns
- Q4-6
- Q8-21

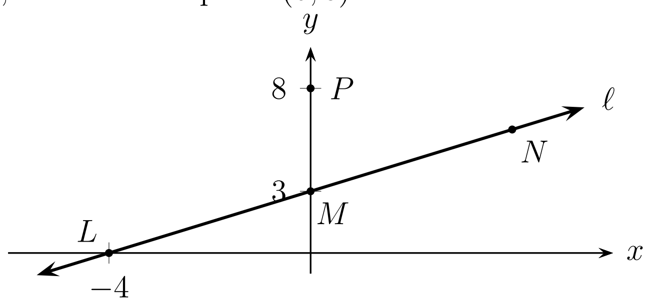
Section D

Legacy HSC questions

D.1 1995 HSC

Question 2

The line ℓ cuts the x axis at $L(-4,0)$ and the y axis at $M(0,3)$ as shown. N is a point on the line ℓ, and P is the point $(0,8)$.



(a) Find the equation of the line l. **2**

(b) Show that the point $(16,15)$ lies on the line ℓ. **1**

(c) By considering the lengths of ML and MP, show that $\triangle LMP$ is isosceles. **2**

(d) Calculate the gradient of the line PL. **1**

(e) M is the midpoint of the interval LN. Find the coordinates of the point N. **2**

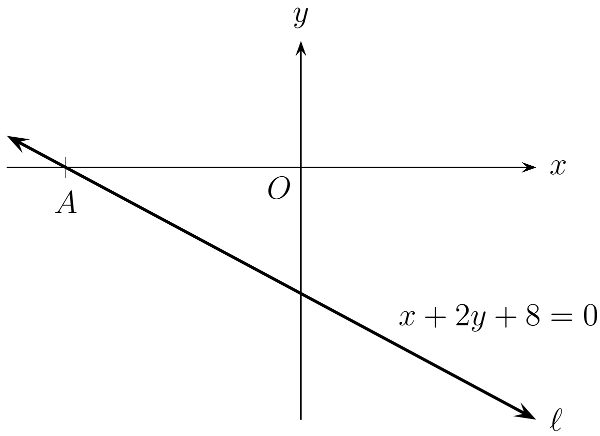
(f) Show that $\angle NPL$ is a right angle. **2**

(g) Find the equation of the circle that passes through the points N, P, and L. **2**

D.2 1996 HSC

Question 2

The line ℓ is shown in the diagram. it has equation $x + 2y + 8 = 0$ and cuts the x axis at A.



The line k has equation $y = -\frac{1}{2}x + 6$, and is not shown on the diagram.

Copy or trace the diagram into your Writing Booklet.

(a) Find the coordinates of A. **1**

(b) Explain why k is parallel to ℓ. **1**

(c) Draw the graph of k on your diagram, indicating where it cuts the axes. **2**

(d) Shade the region $x + 2y + 8 \leq 0$ on your diagram. **1**

(e) Write down a pair of inequalities which define the region between k and l. **2**

(f) Show that $P(8, 2)$ lies on k. **1**

(g) Find the perpendicular distance from P to ℓ. **2**

(h) $Q(4, -6)$ lies on ℓ. Show that Q is the point on ℓ which is closest to P. **2**

D.3 1997 HSC

Question 3

(b) Let A and B be the points $(0, 1)$ and $(2, 3)$ respectively. **6**

i. Find the coordinates of the midpoint of AB.

ii. Find the slope of the line AB.

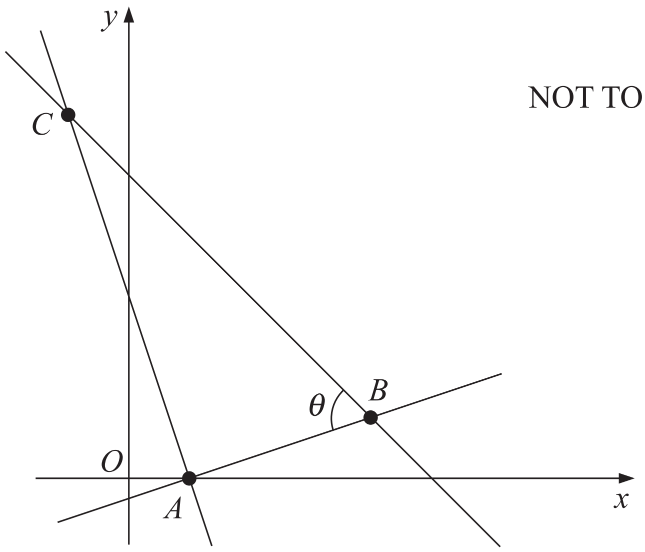
iii. Find the equation of the perpendicular bisector of AB.

iv. The point P lies on the line $y = 2x - 9$ and is equidistant from A and B. Find the coordinates of P.

D.4 1998 HSC

Question 3

The diagram shows points $A(1,0)$, $B(4,1)$ and $C(-1,6)$ in the Cartesian plane. $\angle ABC$ is θ.



NOT TO SCALE

Copy or trace the diagram into your Writing Booklet.

(a) Show that A and C lie on the line $3x + y = 3$. **2**

(b) Show that the gradient of AB is $\frac{1}{3}$. **1**

(c) Show that the length of AB is $\sqrt{10}$ units. **1**

(d) Show that AB and AC are perpendicular. **1**

(e) Find $\tan\theta$. **2**

(f) Find the equation of the circle with centre A that passes through B. **2**

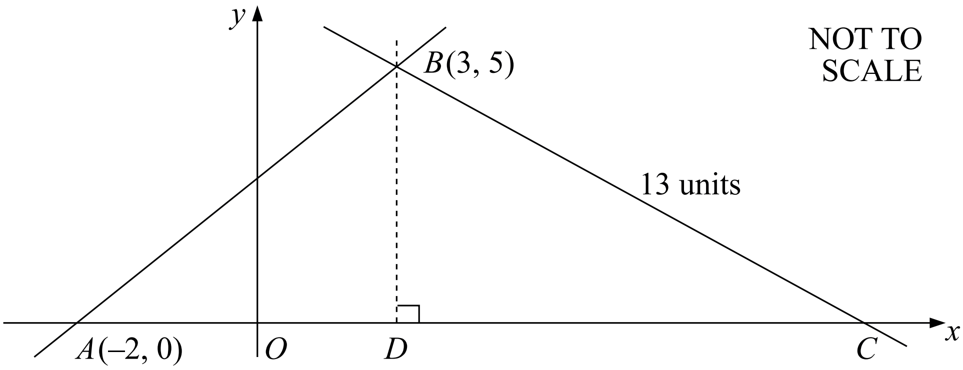
(g) The point D is not shown on the diagram. The point D lies on the line $3x + y = 3$ between A and C,and $AD = AB$. Find the coordinates of D. **2**

(h) On your diagram, shade the region satisfying the inequality $3x + y \leq 3$. **1**

D.5 1999 HSC

Question 2

(b) The diagram shows the points $A(-2,0)$, $B(3,5)$ and the point C which lies on the x axis. The point D also lies on the x axis such that BD is perpendicular to AC. **8**



i. Show that the gradient of AB is 1.

ii. Find the equation of the line AB.

iii. What is the size of $\angle BAC$?

iv. The length of BC is 13 units. Find the length of DC.

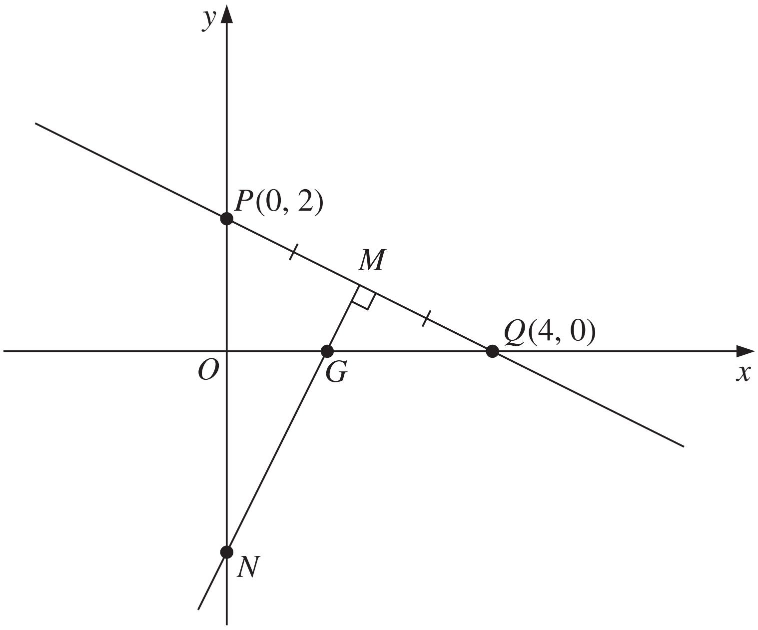
v. Calculate the area of $\triangle ABC$.

vi. Calculate the size of $\angle ABC$, correct to the nearest degree.

D.6 2000 HSC

Question 2

The diagram shows the points $P(0, 2)$ and $Q(4, 0)$. The point M is the midpoint of PQ. The line MN is perpendicular to PQ and meets the x axis at G and the y axis at N.



(a) Show that the gradient of PQ is $-\dfrac{1}{2}$. **1**

(b) Find the coordinates of M. **2**

(c) Find the equation of the line MN. **2**

(d) Show that N has coordinates $(0, -3)$. **1**

(e) Find the distance NQ. **1**

(f) Find the equation of the circle with centre N and radius NQ. **2**

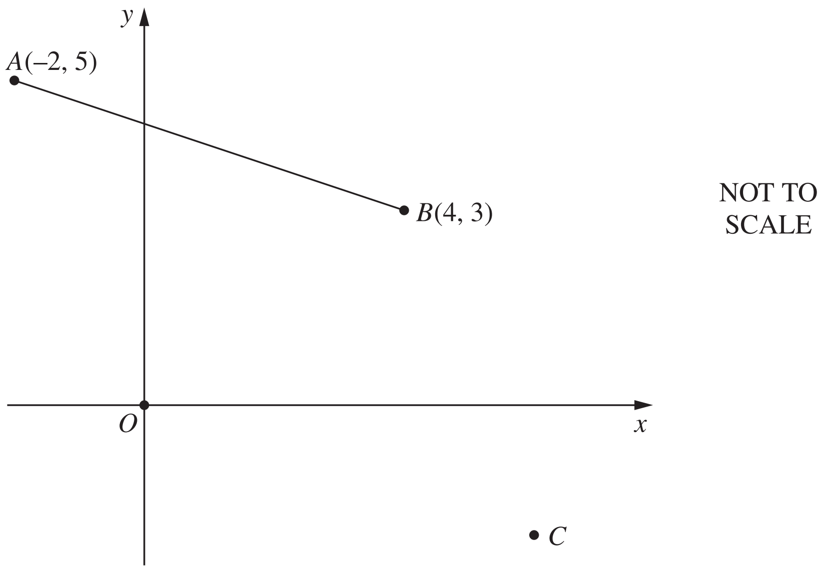
(g) Hence show that the circle in part (f) passes through the point P. **1**

(h) The point R lies in the first quadrant, and $PNQR$ is a rhombus. Find the coordinates of R. **2**

D.7 2001 HSC

Question 2

(b) The diagram shows the points $A(-2, 5)$, $B(4, 3)$ and $O(0, 0)$. The point C is the fourth vertex of the parallelogram $OABC$.



i. Show that the equation of AB is $x + 3y - 13 = 0$. **2**

ii. Show that the length AB is $2\sqrt{10}$. **1**

iii. Calculate the perpendicular distance from O to the line AB. **2**

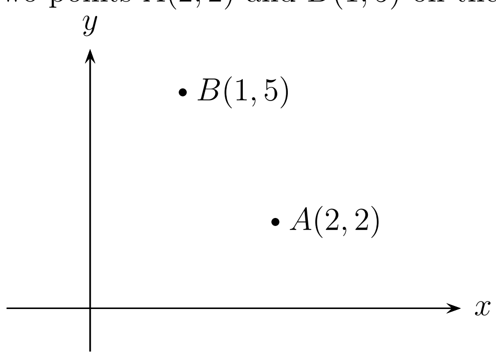
iv. Calculate the area of parallelogram $OABC$. **2**

v. Find the perpendicular distance from O to the line BC. **2**

D.8 2002 HSC

Question 3

(c) The diagram shows two points $A(2, 2)$ and $B(1, 5)$ on the number plane.



Copy the diagram into your writing booklet.

i. Find the coordinates of M, the midpoint of AB. **1**

ii. Show that the equation of the perpendicular bisector of AB is **2**

$$x - 3y + 9 = 0$$

iii. Find the coordinates of the point C that lies on the y axis and is equidistant from A and B. **1**

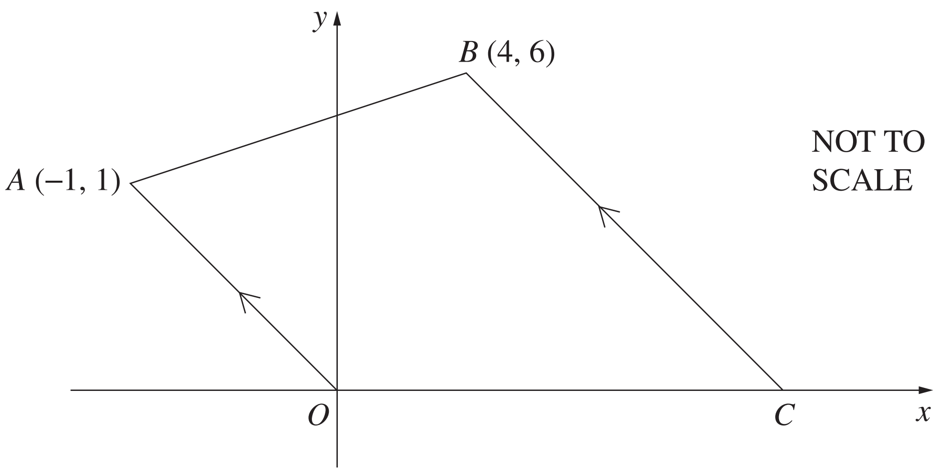
iv. The point D lies on the intersection of the line $y = 5$ and the perpendicular bisector $x - 3y + 9 = 0$. Find the coordinates of D, and mark the position of D on your diagram in your writing booklet. **1**

v. Find the area of $\triangle ABD$. **2**

D.9 2003 HSC

Question 2

(b) In the diagram, $OABC$ is a trapezium with $OA \parallel CB$. The coordinates of O, A and B are $(0, 0)$, $(-1, 1)$ and $(4, 6)$ respectively.



i. Calculate the length of OA. **1**

ii. Write down the gradient of the line OA. **1**

iii. What is the size of $\angle AOC$? **1**

iv. Find the equation of the line BC,and hence find the coordinates of C. **2**

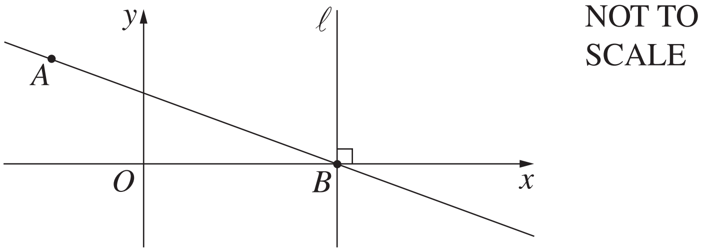
v. Show that the perpendicular distance from O to the line BC is $5\sqrt{2}$. **2**

vi. Hence, or otherwise, calculate the area of the trapezium $OABC$. **2**

D.10 2004 HSC

Question 2

(a) The diagram shows the points $A(-1, 3)$ and $B(2, 0)$. The line ℓ is drawn perpendicular to the x axis through the point B.



i. Calculate the length of the interval AB. **1**

ii. Find the gradient of the line AB. **1**

iii. What is the size of the acute angle between the line AB and the line ℓ? **1**

iv. Show that the equation of the line AB is $x + y - 2 = 0$. **1**

v. Copy the diagram into your writing booklet and shade the region defined by $x + y - 2 \leq 0$. **1**

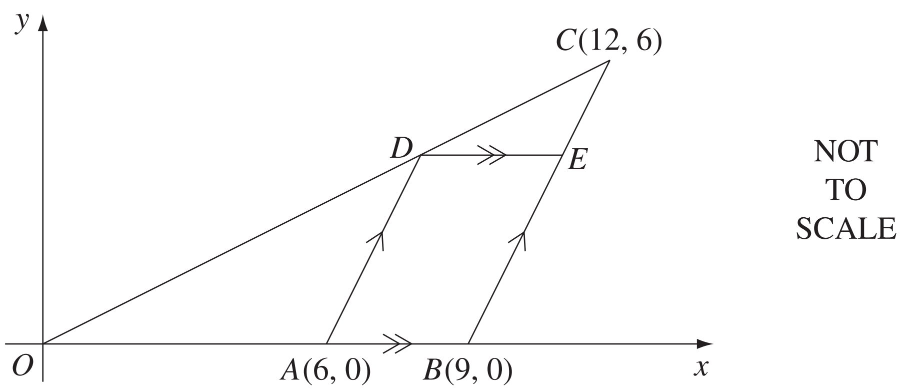
vi. Write down the equation of the line ℓ. **1**

vii. The point C is on the line ℓ such that AC is perpendicular to AB. Find the coordinates of C. **2**

D.11 2005 HSC

Question 3

(c) In the diagram, A, B and C are the points $(6, 0)$, $(9, 0)$ and $(12, 6)$ respectively. The equation of the line OC is $x - 2y = 0$. The point D on OC is chosen so that AD is parallel to BC. The point E on BC is chosen so that DE is parallel to the x axis.



i. Show that the equation of the line AD is $y = 2x - 12$. **2**

ii. Find the coordinates of the point D. **2**

iii. Find the coordinates of the point E. **1**

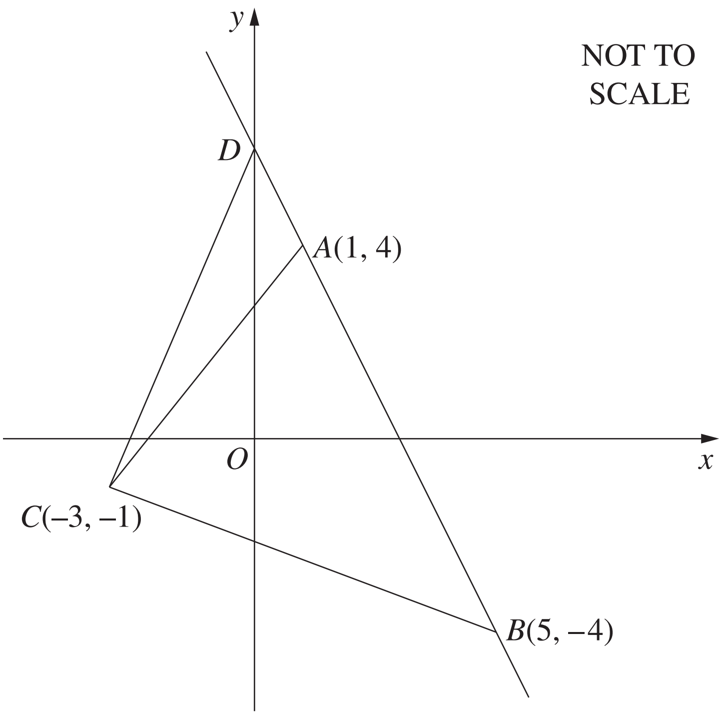
iv. Prove that $\triangle OAD \parallel\!| \triangle DEC$. **2**

v. Hence, or otherwise, find the ratio of the lengths AD and EC. **1**

D.12 2006 HSC

Question 3

(a) In the diagram, A, B and C are the points $(1, 4)$, $(5, -4)$ and $(-3, -1)$ respectively. The line AB meets the y axis at D.



i. Show that the equation of the line AB is $2x + y - 6 = 0$. **2**

ii. Find the coordinates of the point D. **1**

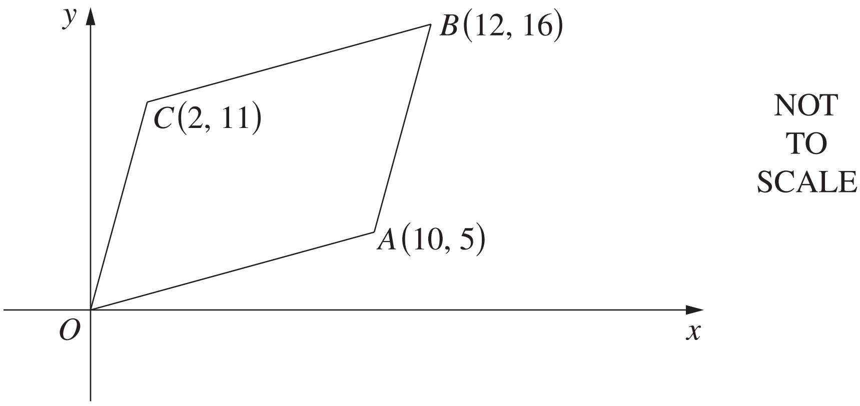
iii. Find the perpendicular distance of the point C from the line AB. **1**

iv. Hence, or otherwise, find the area of the triangle ADC. **2**

D.13 **2007 HSC**

Question 3

(a) In the diagram, A, B and C are the points $(10, 5)$, $(12, 16)$ and $(2, 11)$ respectively.



Copy or trace this diagram into your writing booklet.

i. Find the distance AC. **1**

ii. Find the midpoint of AC. **1**

iii. Show that $OB \perp AC$. **2**

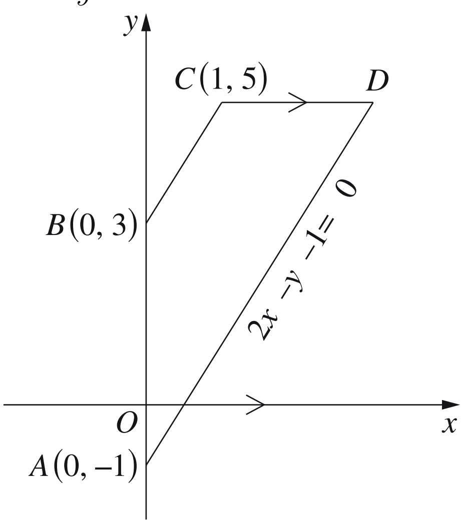
iv. Find the midpoint of OB and hence explain why $OABC$ is a rhombus. **2**

v. Hence, or otherwise, find the area of $OABC$. **1**

D.14 **2008 HSC**

Question 3

(a) In the diagram, $ABCD$ is a quadrilateral. The equation of the line AD is $2x - y - 1 = 0$.



i. Show that $ABCD$ is a trapezium by showing that BC is parallel to AD. **2**

ii. The line CD is parallel to the x axis. Find the coordinates of D. **1**

iii. Find the length of BC. **1**

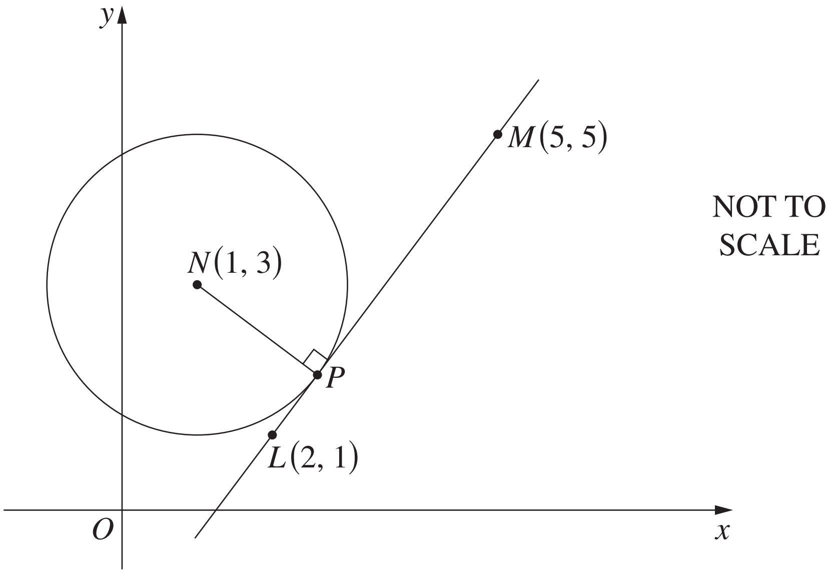
iv. Show that the perpendicular distance from B to AD is $\dfrac{4}{\sqrt{5}}$. **2**

v. Hence, or otherwise, find the area of the trapezium $ABCD$. **2**

D.15 2009 HSC

Question 3

(b) The circle in the diagram has centre N. The line LM is tangent to the circle at P.



i. Find the equation of LM in the form $ax + by + c = 0$. **2**

ii. Find the distance NP. **2**

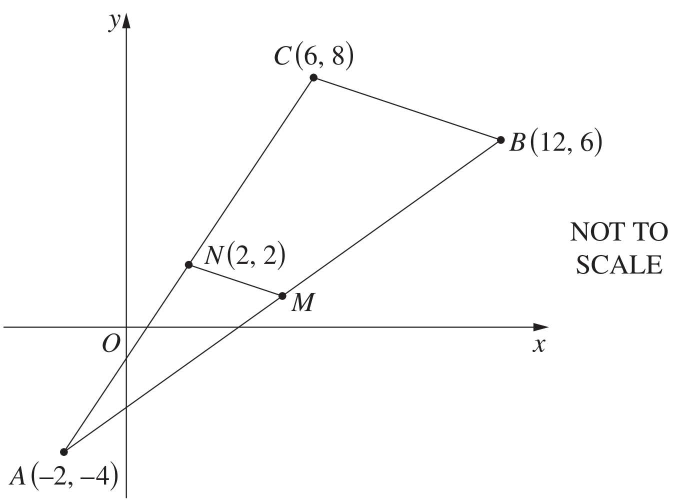
iii. Find the equation of the circle. **1**

(c) Shade the region in the plane defined by $y \geq 0$ and $y \leq 4 - x^2$. **2**

D.16 2010 HSC

Question 3

(a) In the diagram A, B and C are the points $(-2, -4)$, $(12, 6)$ and $(6, 8)$ respectively. The point $N(2, 2)$ is the midpoint of AC. The point M is the midpoint of AB.



i. Find the coordinates of M. **1**

ii. Find the gradient of BC. **1**

iii. Prove that $\triangle ABC$ is similar to $\triangle AMN$. **2**

iv. Find the equation MN. **2**

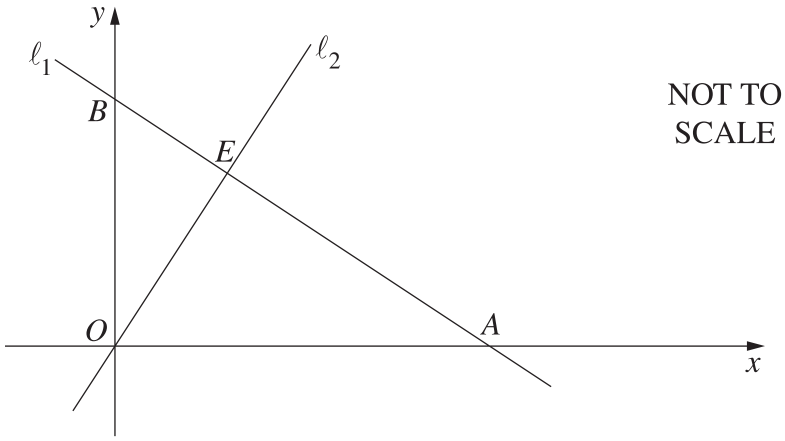
v. Find the exact length of BC. **1**

vi. Given that the area of $\triangle ABC$ is 44 square units, find the perpendicular distance from A to BC. **1**

D.17 2011 HSC

Question 3

(c) The diagram shows a line ℓ_1, with equation $3x+4y-12=0$, which intersects the y axis at B.



A second line ℓ_2, with equation $4x-3y=0$, passes through the origin O and intersects ℓ_1 at E.

i. Show that the coordinates of B are $(0, 3)$. **1**

ii. Show that ℓ_1 is perpendicular to ℓ_2. **2**

iii. Show that the perpendicular distance from O to ℓ_1 is $\dfrac{12}{5}$. **1**

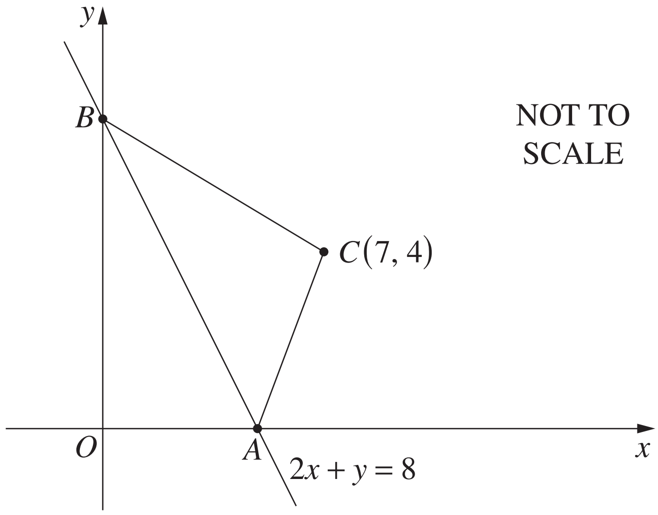
iv. Using Pythagoras' theorem, or otherwise, find the length of the interval BE. **1**

v. Hence, or otherwise, find the area of $\triangle BOE$. **1**

D.18 2012 HSC

Question 13

(a) The diagram shows a triangle ABC. The line $2x + y = 8$ meets the x and y axes at the points A and B respectively. The point C has coordinates $(7, 4)$.



i. Calculate the distance AB. **2**

ii. It is known that $AC = 5$ and $BC = \sqrt{65}$. (Do NOT prove this.) **2**

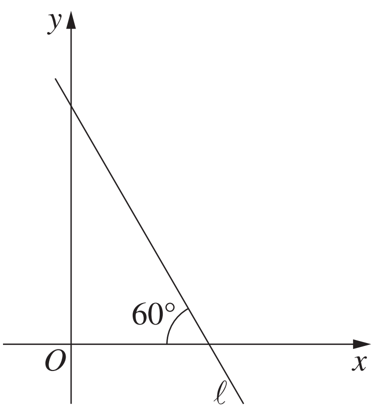
Calculate the size of $\angle ABC$ to the nearest degree.

iii. The point N lies on AB such that CN is perpendicular to AB. **3**

Find the coordinates of N.

D.19 2013 HSC

2. The diagram shows the line ℓ. **1**



(A) $\sqrt{3}$

(B) $-\sqrt{3}$

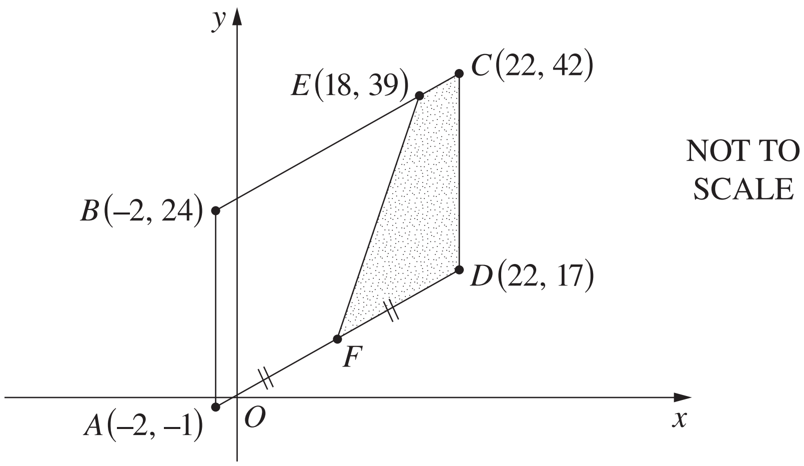
(C) $\dfrac{1}{\sqrt{3}}$

(D) $-\dfrac{1}{\sqrt{3}}$

What is the slope of line ℓ?

Question 12

(b) The points $A(-2,-1)$, $B(-2,24)$, $C(22,42)$ and $D(22,17)$ form a parallelogram as shown. The point $E(18,39)$ lies on BC. The point F is the midpoint of AD.



i. Show that the equation of the line through A and D is $3x-4y+2=0$. **2**

ii. Show that the perpendicular distance from B to the line through A and D is 20 units. **1**

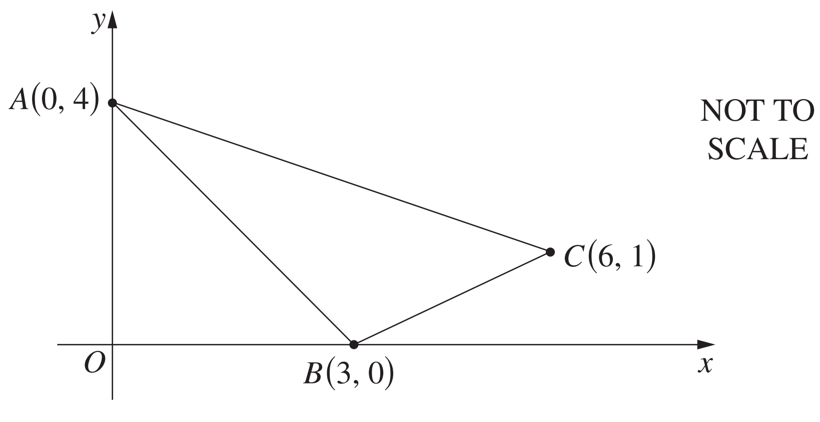
iii. Find the length of EC. **1**

iv. Find the area of the trapezium $EFDC$. **2**

D.20 2014 HSC

Question 12

(b) The points $A(0,4)$, $B(3,0)$ and $C(6,1)$ form a triangle, as shown in the diagram.



i. Show that the equation of AC is $x+2y-8=0$. **2**

ii. Find the perpendicular distance from B to AC. **2**

iii. Hence, or otherwise, find the area of $\triangle ABC$. **2**

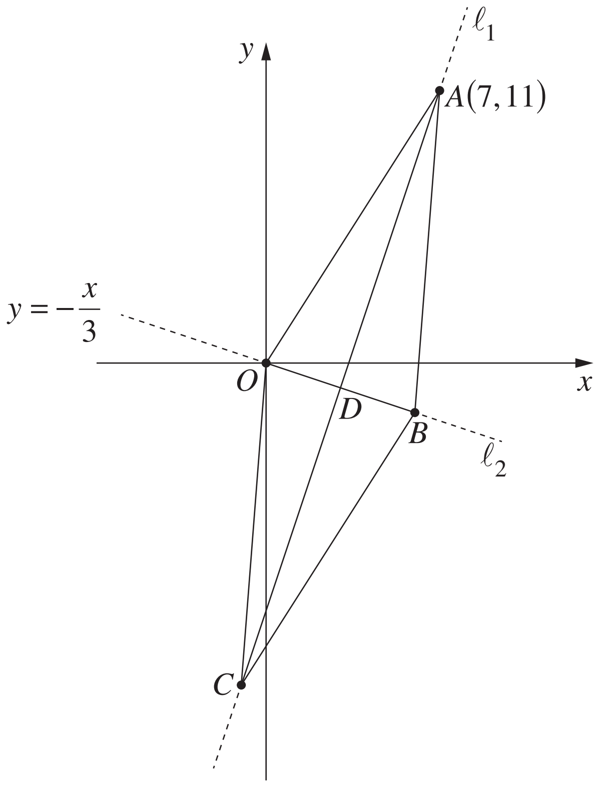
D.21 2015 HSC

Question 12

(b) The diagram shows the rhombus $OABC$.

The diagonal from the point $A(7, 11)$ to the point C lies on the line ℓ_1.

The other diagonal, from the origin O to the point B, lies on the line ℓ_2 which has equation $y = -\dfrac{x}{3}$.



i. Show that the equation of the line ℓ_1 is $y = 3x - 10$. **2**

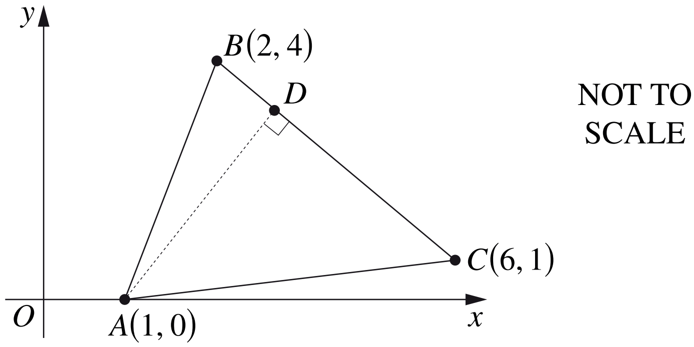
ii. The lines ℓ_1 and ℓ_2 intersect at the point D. **2**

Find the coordinates of D.

D.22 2016 HSC

Question 12

(a) The diagram shows points $A(1,0)$, $B(2,4)$ and $C(6,1)$. The point D lies on BC such that $AD \perp BC$.

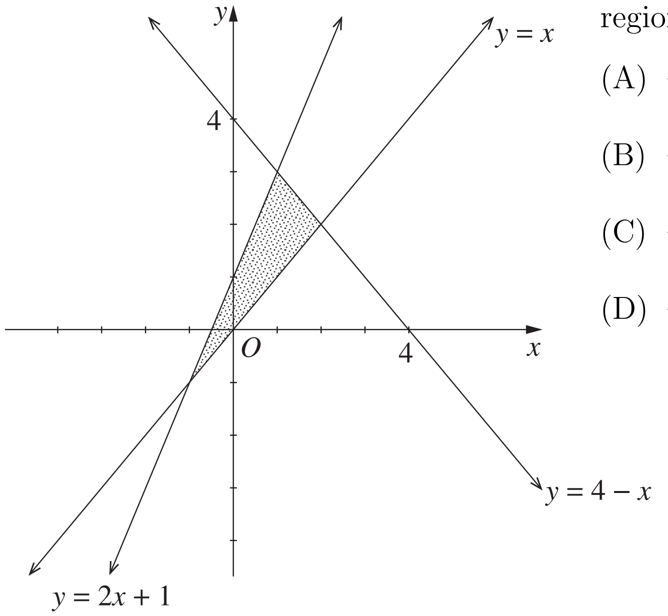


i. Show that the equation of BC is $3x + 4y - 22 = 0$. **2**

ii. Find the length of AD. **2**

iii. Hence or otherwise, find the area of $\triangle ABC$. **2**

4. The region enclosed by $y = 4 - x$, $y = x$ and $y = 2x + 1$ is shaded in the diagram. **1**



Which of the following defines the shaded region?

(A) $y \le 2x + 1, \quad y \le 4 - x, \quad y \ge x.$

(B) $y \ge 2x + 1, \quad y \le 4 - x, \quad y \ge x.$

(C) $y \le 2x + 1, \quad y \ge 4 - x, \quad y \ge x.$

(D) $y \le 2x + 1, \quad y \ge 4 - x, \quad y \ge x.$

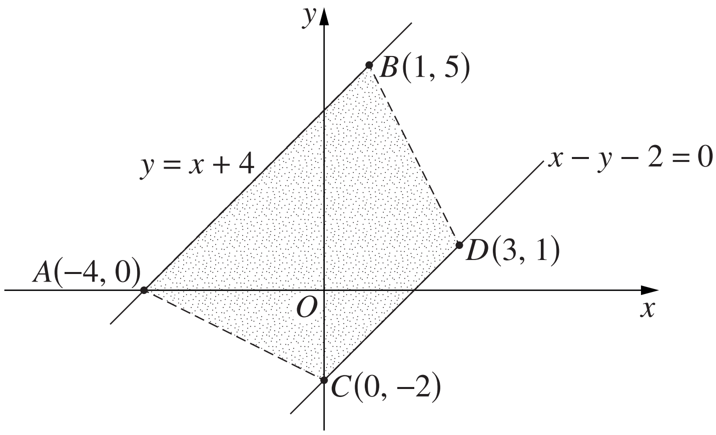
Question 12

(c) The points $A(-4, 0)$ and $B(1, 5)$ lie on the line $y = x + 4$.

The length of AB is $\sqrt{2}$.

The points $C(0, -2)$ and $D(3, 1)$ lie on the line $x - y - 2 = 0$.

The points A, B, D, C form a trapezium as shown.



NOT TO SCALE

i. Find the perpendicular distance from the point $A(-4, 0)$ to the line $x - y - 2 = 0$. **1**

ii. Calculate the area of the trapezium. **2**

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1},\ r \neq 1$$

$$S = \frac{a}{1-r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

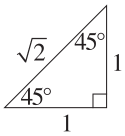
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \dfrac{\text{opp}}{\text{hyp}}, \quad \cos A = \dfrac{\text{adj}}{\text{hyp}}, \quad \tan A = \dfrac{\text{opp}}{\text{adj}}$

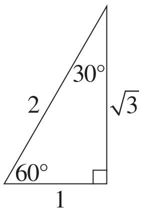
$A = \dfrac{1}{2}ab\sin C$



$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$



$l = r\theta$

$A = \dfrac{1}{2}r^2\theta$

Trigonometric identities

$\sec A = \dfrac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \dfrac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \dfrac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\dfrac{A}{2}$ then $\sin A = \dfrac{2t}{1 + t^2}$

$\cos A = \dfrac{1 - t^2}{1 + t^2}$

$\tan A = \dfrac{2t}{1 - t^2}$

$\cos A\cos B = \dfrac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A\sin B = \dfrac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A\cos B = \dfrac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A\sin B = \dfrac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \dfrac{1}{2}(1 - \cos 2nx)$

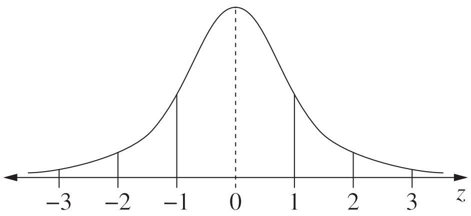
$\cos^2 nx = \dfrac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \dfrac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$E(X) = \mu$

$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \leq x) = \displaystyle\int_a^x f(x)\,dx$

$P(a < X < b) = \displaystyle\int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r (1 - p)^{n - r}$

$X \sim \operatorname{Bin}(n, p)$

$\Rightarrow \ P(X = x)$

$= \dbinom{n}{x} p^x (1 - p)^{n - x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\operatorname{Var}(X) = np(1 - p)$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_r = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$